1. Minimize $3x^2 + 3y^2$, subject to the constraint $150 - 3x - 4y = 0$.

   The minimum value of the function is $\square$.
   (Type an exact answer in simplified form.)

2. Let $f(x,y) = x^2 + y^2 - 2x + 2y + 2$.

   1. Find all points $(x,y)$ where $f(x,y)$ has a possible relative maximum or minimum.
   2. Using second derivative test to determine if they are maximum or minimum.

3. Determine the average value of $f(x)$ over the interval from $x = a$ to $x = b$, where $f(x) = \frac{1}{x}$, $a = \frac{1}{7}$, and $b = 7$.

   The average value is $\square$.
   (Type an exact answer in simplified form.)

4. Calculate the definite integral.

   $\int_{1}^{4} 7 \, dx$

   $\int_{1}^{4} 7 \, dx = \square$ (Simplify your answer.)

5. Calculate the definite integral.

   $\int_{0}^{4} -8e^{5x} \, dx$

   $\int_{0}^{4} -8e^{5x} \, dx = \square$ (Type an exact answer in simplified form.)

6. Evaluate the definite integral.

   $\int_{0}^{4} (5x^2 - 8x + 2) \, dx$

   $\int_{0}^{4} (5x^2 - 8x + 2) \, dx = \square$
   (Type an integer or a simplified fraction.)
7. Determine the following.

\[ \int \left( \frac{12}{\sqrt{x}} + 12\sqrt{x} \right) \, dx \]

\[ \int \left( \frac{12}{\sqrt{x}} + 12\sqrt{x} \right) \, dx = \square \]

(Use C as the arbitrary constant. Simplify your answer.)

8. Find the indefinite integral \( \int \left( x - 2x^7 + \frac{1}{5x} \right) \, dx \).

\[ \int \left( x - 2x^7 + \frac{1}{5x} \right) \, dx = \square \]

(Simplify your answer. Use C as an arbitrary constant. Use integers or fractions for any numbers in the expression.)

9. Find the derivatives of the following functions:

1. \( f(x) = e^{e^x} \)
   \[ f'(x) = e^{e^x} \cdot e^x \]

2. \( f(x) = 2 \sqrt{x} \)
   \[ f'(x) = \frac{2}{\sqrt{x}} \ln 2 \cdot \frac{1}{2 \sqrt{x}} = -2 \sqrt{x} \ln 2 \cdot \frac{1}{2 \sqrt{x}} \]

3. \( f(x) = x \ln x \)
   \[ f'(x) = 1 + \ln x \]

Find \( \frac{\partial f}{\partial x} \) and \( \frac{\partial^2 f}{\partial x^2} \), \( \frac{\partial^2 f}{\partial y^2} \), \( \frac{\partial^2 f}{\partial x \partial y} \) of the following functions

4. \( f(x) = (x + y^2)^3 \)
   \[ \frac{\partial f}{\partial x} = 3(x + y^2)^2 \cdot 2x \]
   \[ \frac{\partial f}{\partial y} = 3(x + y^2)^2 \cdot 2y \]
   \[ \frac{\partial^2 f}{\partial x^2} = 6(x + y^2) \cdot 2x \]
   \[ \frac{\partial^2 f}{\partial y^2} = 12(x + y^2) \cdot 2y \]
   \[ \frac{\partial^2 f}{\partial x \partial y} = 12y(x + y^2) \]

5. \( f(x) = 2x^3y \)

6. \( f(x) = \frac{x}{y - 2} \)

\[ \frac{\partial f}{\partial x} = \frac{y - 2}{2} \]

\[ \frac{\partial f}{\partial y} = \frac{x}{(y - 2)^2} \]

\[ \frac{\partial^2 f}{\partial x^2} = \frac{-1}{(y - 2)^2} \]

\[ \frac{\partial^2 f}{\partial y^2} = \frac{-2x}{(y - 2)^3} \]

\[ \frac{\partial^2 f}{\partial x \partial y} = \frac{2}{(y - 2)^3} \]
10. Calculate the following definite and indefinite integrals.

1. \(\int e^{3x} + \frac{1}{x} \, dx = \frac{e^{3x}}{3} + \ln |x|\)

2. \(\int 2^{x+1} \, dx = 2 \int 2^x \, dx = \frac{2}{\ln 2} \cdot 2^x\)

3. \(\int \frac{e^x + e^{-x}}{2} \, dx = \frac{1}{2} (e^x - e^{-x})\)

4. \(\int_0^{\ln 3} e^{-2t} \, dt = -\frac{1}{2} \left[ \frac{e^{-2t}}{-2} \right]_0^{\ln 3} = -\frac{1}{2} \left( \frac{1}{3} - 1 \right) = \frac{1}{6}\)

5. \(\int_0^{1} e^{-x} \, dx = \left. -e^{-x} \right|_0^1 = -\left( e^{-1} - 1 \right)\)

6. \(\int_1^2 \left( x^2 + \frac{1}{x} \right)^2 \, dx = \int_1^2 \left( x^4 + 2 + 2x + \frac{2}{x^2} + \frac{1}{x} \right) \, dx = \frac{x^5}{5} \bigg|_1^2 + \frac{1}{x} \bigg|_1^2 + x^2 \bigg|_1^2\)

11. Find the area of the region enclosed by the functions. \(y = \frac{2x(x^2 - 4)}{5}\) and \(y = 2x\).

\[ f(x) = \frac{2x(x^2 - 4)}{5} = 2x \implies \text{Intersection points are } x = 0 \quad \text{and } x = \pm 3 \]

\[ \int_{-3}^{3} |f(x) - g(x)| \, dx = \int_{-3}^{0} \left( \frac{2x(x^2 - 4)}{5} - 2x \right) + \int_{0}^{3} \left( 2x - \frac{2x(x^2 - 4)}{5} \right) \, dx \]

12. A rectangular closed box is to be built at minimum cost to hold 64 cubic inches. Since the cost will depend on the surface area, find the dimensions that will minimize the surface area of the box.

The dimensions that will minimize the surface area of the box are \(\square\) inches.

(Simplify your answer. Use a comma to separate answers as needed.)

13. Let \(f(x)\) be a function satisfying the differential equation \(\frac{df(x)}{dx} = f(x) + \ln x - \frac{1}{x}\).

1) Let \(g(x) = \int \frac{1}{x} \, dx\), find the equation that \(f(x) + g(x)\) must satisfy.

2) Solve the differential equation for \(f(x) + g(x)\) you obtained from the part 1).

3) Find all solutions of the differential equation \(\frac{df(x)}{dx} = f(x) + \ln x - \frac{1}{x}\).
1. \( f(x, y) = 3x^2 + 3y^2, \quad g(x, y) = 150 - 3x - 4y \)

\[
F(x, y) = f(x, y) + g(x, y) = 3x^2 + 3y^2 + \lambda (150 - 3x - 4y)
\]

\[
\begin{align*}
0 &= \frac{\partial F}{\partial x} = 6x - 3\lambda \\
0 &= \frac{\partial F}{\partial y} = 6y - 4\lambda \\
0 &= \frac{\partial F}{\partial \lambda} = 150 - 3x - 4y
\end{align*}
\]

\[
\begin{align*}
x &= \frac{\lambda}{2} \\
y &= \frac{\lambda}{3}
\end{align*}
\]

\[
\begin{align*}
\lambda &= 36 \\
x &= 18 \\
y &= 24
\end{align*}
\]

\[
\min f(x, y) = 3(18)^2 + 3(24)^2
\]

\[g(x, y) = 0\]

2. \( f(x, y) = x^2 + y^2 - 2x + 2y + 2 \)

1) \[
\frac{\partial f}{\partial x} = 2x - 2 = 0 \quad y = -1
\]

\[
\frac{\partial f}{\partial y} = 2y + 2 = 0 \quad x = 1
\]

2) \[
\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = 2, \quad \frac{\partial^2 f}{\partial x \partial y} = 0
\]

\[
D(x, y) = \left( \frac{\partial^2 f}{\partial x^2} \right) \left( \frac{\partial^2 f}{\partial y^2} \right) - (\frac{\partial^2 f}{\partial x \partial y})^2 = 2 > 0
\]

\[
\begin{align*}
\frac{\partial^2 f}{\partial x^2} \bigg|_{(x=1, y=-1)} &= 2 > 0 \quad \text{Local max.}
\end{align*}
\]
\[
\int_{-3}^{0} \left( \frac{2x(x^2-9)-2x}{5} \right) \, dx
\]

\[
= \frac{1}{5} \int_{-3}^{0} (2x^3 - 8x) \, dx - \int_{-3}^{0} 2x \, dx
\]

\[
= \frac{1}{5} \left( \frac{x^4}{2} - 4x^2 \right) \bigg|_{-3}^{0} - \left. x^2 \right|_{-3}^{0}
\]

\[
= 0 - \frac{1}{5} \left( \frac{81}{2} - 36 \right) - (0 - 9)
\]

\[
= 9 - \frac{9}{10} = \frac{81}{10}
\]

\[
\int_{0}^{3} \left( 2x - \frac{2x(x^2-4)}{5} \right) \, dx
\]

\[
= \frac{81}{10} \quad \text{(check!)}
\]

12. \( G4= \text{Volume} = x \cdot y \cdot z \quad z = \frac{64}{xy} \)

\( A(x,y)= \text{Area} = 2xy + 2yz + 2xz \)

\[
= 2 \left( xy + y \left( \frac{64}{xy} \right) + x \left( \frac{64}{xy} \right) \right)
\]

\[
= 2 \left( xy + \frac{64}{x} + \frac{64}{y} \right)
\]

\[
\frac{\partial A}{\partial x} = 2(y - \frac{64}{x^2}) = 0 \quad \Rightarrow \quad x^2y = 64 \quad \Rightarrow \quad y = \frac{64}{x^2}
\]

\[
\frac{\partial A}{\partial y} = 2(x - \frac{64}{y^2}) = 0 \quad \Rightarrow \quad xy^2 = 64
\]

\[
\Rightarrow \quad xy^2 = x \cdot \left( \frac{64}{x^2} \right)^2 = 64 \quad \Rightarrow \quad x = 4
\]

\[
\Rightarrow \quad \frac{64}{x^3} = 1
\]