What does the functional equation of the Riemann Zeta function have to do with the proof of Fermat’s Last Theorem? In both cases one starts with an arithmetic object (the integers in the first case, an elliptic curve over $\mathbb{Q}$ in the second) and links it to an analytic function with surprising properties. These are special cases of a much larger conjectural story (within the Langlands program) which predicts such a construction for all arithmetic varieties. In this colloquium talk, we discuss some of the impressive advances that have been made on this problem in the last few years.